



Use of Reversion Method to Solve Mathematical Models on Series Electrical Circuits (RL) with Nonlinear Inductors

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Abstract. Forming a mathematical model of a series electrical circuit with a nonlinear inductor obtained based on Kirchoff's laws one and two takes the form of a nonlinear differential equation. Approach methods are used to determine the electric current in the circuit, including the Reversion Method, namely by converting nonlinear differential equations into a system of linear differential equations.

Keywords: reversion method, series electrical circuit, inductor, nonlinear

INTRODUCTION

In today's rapid development of science, especially in technological advances, scientists are required to be able to solve problems arising from these technological advances. Applications of mathematics, especially differential equations, are widely used in almost all fields of science, such as engineering and industry, physics, management, astronomy, psychology, economics, engineering, and many others, because mathematics is the basis of science to solve problems arising from the rapid progress of science and technology today.

The application of mathematics in the field of electrical engineering is the application of the reversion method in series electrical circuits with nonlinear inductors where the mathematical model of the electric circuit is in the form of a nonlinear differential equation whose online is caused by changes in magnetically induced flux $F(t)$ in the coil so that at both ends of the coil there will be an induced or impacted electromotive force (gg).

The reversion method is used to solve the mathematical model of the electrical circuit, which is an approach method that converts nonlinear differential equations into a system of linear differential equations. In solving the system of linear differential equations obtained, the Laplace transformation method should be used because it saves much numerical work.

METHOD

This method is based on an algebraic procedure that converts a nonlinear differential equation into a system of linear differential equations.

Suppose that the desired differential equation for the solution takes the form :

$$a_1y + a_2y^2 + a_3y^3 + a_4y^4 + \dots + a_7y^7 + \dots = k\Phi(t) \quad (1)$$

where a_i is usually a function of the operator D with $D = d/s$ and $a_1 \neq 0$ and it is assumed that a solution takes the form :

$$y = A_1k + A_2k^2 + A_3k^3 + A_4k^4 + \dots \quad (2)$$

to determine the coefficients A_i that is, by substituting equation (2) into (1) and equating the power coefficients k . In the same way, if this is done, then coefficients are obtained $A_1, A_2, A_3, A_4, \dots$ and so on.

In this writing only displays seven coefficients A_i . The first is :

$$A_1 = \Phi(t)/a_1 \quad (3)$$

$$A_2 = -a_2/a_1 \cdot A_1^2 \quad (4)$$

$$A_3 = -1/a_1(2a_2A_1A_2 + a_3A_1^3) \quad (5)$$

$$A_4 = -1/a_1(a_2(2A_1A_3 + A_2^2) + 3a_3A_1^2A_2 + a_4A_1^4) \quad (6)$$

$$A_5 = -1/a_1(a_2(2A_1A_4 + 2A_2A_3) + a_3(3A_1^2A_3 + 3A_2^2A_1) + 4a_4A_1^3A_2 + a_5A_1^5) \quad (7)$$

$$A_6 = -1/a_1(a_2(2A_1A_5 + 2A_2A_4 + A_3^2) + a_3(3A_1^2A_4 + A_2^3 + 6A_1A_2A_3) + a_4(4A_1^3A_3 + 6A_1^2A_2^2) + 5a_5A_1^4A_2 + a_6A_1^6) \quad (8)$$

$$A_7 = -1/a_1(a_2(2A_1A_6 + 2A_2A_5 + 2A_3A_4) + a_3(3A_1^2A_5 + 3A_2^2A_3 + 3A_3^2A_1 + 6A_1A_2A_4) + a_4(4A_1^3A_4 + 4A_2^3A_1 + 12A_1^2A_2A_3) + a_5(5A_1^4A_3 + 5A_1^3A_2^2) + 6a_6A_1^5A_2 + a_7A_1^7) \quad (9)$$

Series Electrical Circuit with Nonlinear Inductor

Kirchoff's law has a very important role in the formation of electrical circuit equations. Kirchoff's first law or branch point law reads: the algebraic number of currents entering a branch point of a network is zero, that is:

$$\sum i = 0 \quad (10)$$

while Kirchoff's law, better known as the law of loops, reads the algebraic sum of electromotive forces (ggl) in each circuit loop, equal to the algebraic sum of iR products in the same loop i.e. :

$$\sum E = \sum iR \quad (11)$$

Take a look at figure 1 below :

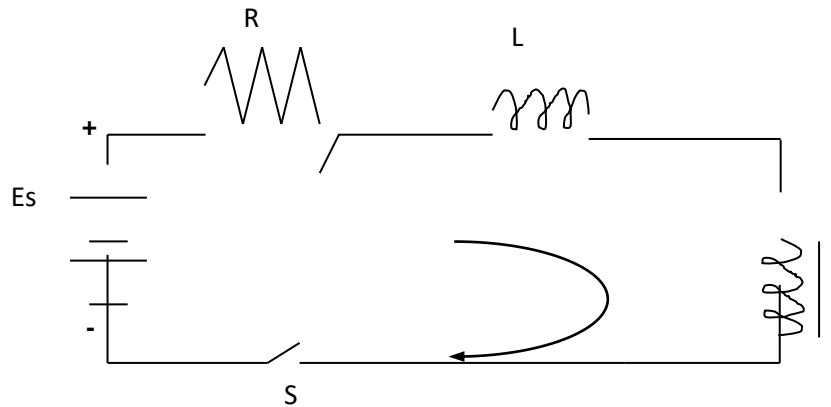


Figure 1. Series electrical circuit with nonlinear inductor

This circuit consists of an RL circuit along with a nonlinear inductor. Suppose that a coil or coil of wire consists of N windings and if the switch S is closed, the current in the circuit will change di/dt which is large, further this will cause a change in the magnetic induction flux $\Phi(t)$ inside the coil, so that at both ends of the coil there will be electromotive force (ggl) induced or impacted by :

$$E_L = -N \frac{d\Phi(t)}{dt} \quad (12)$$

flux changes $N \frac{d\Phi(t)}{dt}$ can be written as :

$$E_L = -N \frac{d\Phi(t)}{dt} = -N \frac{d\Phi(t)}{di} \cdot \frac{di}{dt} \quad (13)$$

Defined self-inductance as :

$$E_L = N \frac{d\Phi(t)}{di} \quad (14)$$

the unit for self-inductance is Hendry (H).

A coil that is made to have a certain price is called an *inductor*.

If the equation (14) substituted into equations (13) then obtained

$$E_L = -L \frac{di}{dt} \quad (15)$$

Here L is an inductance inductance, according to Lenz's law the direction of electromotive force (ggl) this induction will counteract the cause.

To determine the current equation in the above electrical circuit can be used Kirchoff's law as follows :

$\sum iR = \sum E$ So that the differential equation in current is obtained, namely: :

$$L \frac{di}{dt} + iR + \frac{d\Phi(t)}{dt} = E_S ; t > 0 \quad (16)$$

with the switch assumed closed on $t = 0$ so that $i = 0$ at $t = 0$ where $\Phi(t)$ is the total flux connection of a nonlinear inductor and is generally a function of i depending on the material of the inductor. A typical form of analytics assumed for functional relationships between fluxes $\Phi(t)$ and the current i is :

$$\Phi(t) = c_1 i - c_2 i^3 \quad (17)$$

with c_1 and c_2 is the specified setting of the nutility curve of the core ferromagnetic material.

The above electrical circuit can be developed for a general circuit in series with a nonlinear inductor, as in figure 2 below :

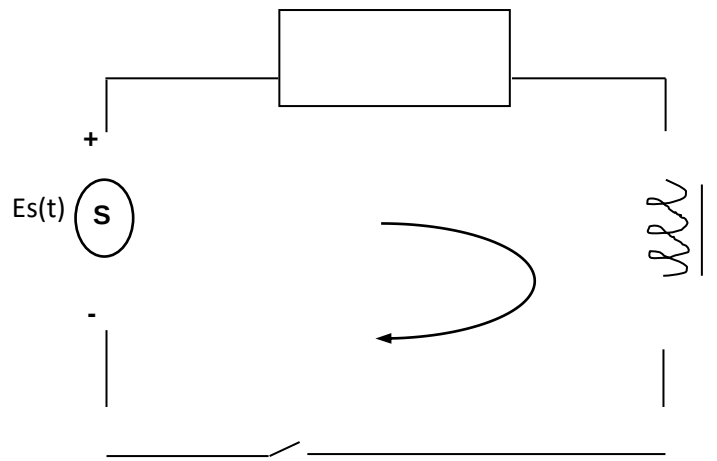


Figure 2. Common circuits in series with nonlinear inductors

If the above circuit has an operational impedance $P(D)$ where $D = d/dt$ and using Kirchoff's law, the second differential equation for current is obtained: :

$$P(D)i + d\phi(t)/dt = Es(t) ; D = d/dt \quad (18)$$

with $Es(t)$ is the electromotive force (ggl) which is imposed as a function of time.

It is assumed that the switch closes on $t = 0$ so that $i = 0$ at $t = 0$, and generally flux $\phi(t)$ be GASAL's function with a series is as follows: :

$$\phi(t) = c_1 i + c_3 i^3 + c_4 i^4 + \dots = \sum_{n \text{ ganjil}} c_n i^n \text{ where } c_n \text{ is the setting} \quad (19)$$

DISCUSSION

From the description above obtained the form of differential equations:

For series electrical circuits (RL) with nonlinear inductors.

The form of the differential equation :

$$L \frac{di}{dt} + iR + d\Phi(t)/dt = E_s \quad ; \quad t > 0 \quad (20)$$

$$\text{where } : \Phi(t) = c_1 i - c_2 i^3 \quad (21)$$

with IC : $i = 0$ at $t = 0$

if the equation (21) substituted equation (20) Obtained :

$$L \frac{di}{dt} + iR + d(c_1 i - c_2 i^3)/dt = E_s \quad (22)$$

or stated in the operator $D = d/dt$ Obtained :

$$(L + c_1)Di + iR - c_2 Di^3 = E_s \quad (23)$$

$$\text{further e.g. } b = R/(L+c_1) \quad (24)$$

Thus the equation (23) can be written in the form of :

$$(D + b)i - bc_2/R Di^3 = E_s b/R \quad (25)$$

If the equation (25) Compared to equations (1) obtained the following values :

$$a_1 = D + b \quad (26)$$

$$a_2 = 0 \quad (27)$$

$$a_3 = -bc_2 D/R \quad (28)$$

$$k = 1 \quad (29)$$

$$\Phi(t) = E_s b/R \quad (30)$$

Differential equation for A_1 from the equation (3) and by substituting equations (26) and (30)

Retrieved :

$$(d/dt + b)A_1 = E_s b/R \quad (31)$$

Equation (31) is a first-order linear differential equation which, if solved, is obtained :

$$A_1 = E_s/R (1 - e^{-bt}) \quad (32)$$

Differential equation for A_2 from the equation (4) and by substituting equations (26), (27), and (32) Obtained:

$$A_2 = 0 \quad (33)$$

Differential equation for A_3 from the equation (5) and by substituting equations (26), (27), (28), (32), and (33) Obtained:

$$(d/dt + b)A_3 = bc_2Es^3/R^4 (3be^{-bt} - 6b e^{-2bt} + 3be^{-3bt}) \quad (34)$$

Equation (34) is a first-order linear differential equation which, if solved, is obtained :

$$A_3 = bc_2Es^3/R^4 (-9/2e^{-bt} + 3bt e^{-bt} + 6e^{-2bt} - 3/2 e^{-3bt}) \quad (35)$$

to A_4 , A_5 , and so can be found in the same way as above.

So the current in the equation (20) given by equation :

$$\begin{aligned} i &= A_1 + A_3 + \dots \quad \text{or} \\ i &= Es/R (1 - e^{-bt}) + bc_2Es^3/R^4 (-9/2e^{-bt} + 3bt e^{-bt} + 6e^{-2bt} - 3/2 e^{-3bt}) + \dots \end{aligned} \quad (36)$$

For series general electrical circuits (RL) with nonlinear inductors.

The form of the differential equation :

$$P(D)i + d\phi(t)/dt = Es(t) \quad ; \quad D = d/dt \quad ; \quad \text{and } t > 0 \quad (37)$$

$$\text{with } \phi(t) = \sum_{n \text{ ganjil}} c_n i^n \quad \text{where } c_n \text{ is the setting} \quad (38)$$

with IC : $i = 0$ at $t = 0$

If the equation (38) substituted equation (37) Obtained :

$$\begin{aligned} P(D)i + \sum c_n Di^n &= Es(t) \quad ; \quad \text{with } D = d/dt \quad \text{and } n \text{ odd or} \\ (P(D) + c_1D)i + \sum c_m Di^m &= Es(t) \quad ; \quad \text{with } D = d/dt \quad \text{and } m = 3, 5, 7, \dots \end{aligned} \quad (39)$$

If the equation (39) Compared to equations (1) obtained the following values :

$$a_1(D) = P(D) + c_1D \quad (40)$$

$$a_m = \begin{cases} c_m D & ; \quad m = 3, 5, 7, \dots \end{cases} \quad (41)$$

$$a_m = \begin{cases} 0 & ; \quad m = 2, 4, 6, \dots \end{cases} \quad (42)$$

$$\Phi(t) = Es(t) \quad (43)$$

Differential equation for A_1 from the equation (3) and by substituting equations (40) and (43)

Retrieved :

$$(P(D) + c_1D)A_1 = Es(t) \quad (44)$$

or by using the Laplace Transformation Method obtained :

$$((P(D) + c_1 D)A_1) = (Es(t)) \quad (45)$$

$$\text{If Laplace Transform } (Es(t)) = Es^{-1}(s) \quad (46)$$

$$\text{and } (A_1(t)) = A_1^{-1}(s) \quad (47)$$

then the transformation of the Laplace equation (47) be:

$$A_1^{-1}(s) = Es^{-1}(s)/(P(s)+c_1 s) = Es^{-1}(s)/a_1(s) \quad (48)$$

$$\text{Jadi } A_1(t) = (A_1^{-1}(s)) = (Es^{-1}(s)/a_1(s)) \quad (49)$$

To continue that settlement, it is now necessary to count the rows A_2 , A_3 , and so on,

because $a_2 = 0$ from the equation (44) then value $A_2 = 0$.

Differential equation for A_3 be :

$$A_3 = -a_3/a_1 \cdot A_1^3 \quad (50)$$

With the same procedure as above, it will be obtained :

$$A_3^{-1}(s) = -a_3(s)/a_1(s) \quad (51)$$

For value $A_4 = 0$ because $a_2 = a_4 = A_2 = 0$ and so on.

Thus the current in the network is given by :

$$i = A_1 + A_3 + A_5 + \dots \quad (52)$$

CONCLUSION

The Reversion method is one method to solve nonlinear differential equations, namely by converting nonlinear differential equations into a system of linear differential equations. The Laplace Transform Method makes it easier to solve linear differential equations. A mathematical model of a series electrical circuit with a nonlinear inductor in the form of a nonlinear differential equation. The case solved by this reversion method is a series electrical circuit (RL) with a nonlinear inductor and a typical circuit in series with a nonlinear inductor whose result is numeric.

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